

✓ Experiment: Any action or process to prove or disprove doubtful is called an experiment.

Trial: The single performance of an experiment is called a trial.

✓ Random Experiment: An experiment which produces different results even though it is repeated under the identical conditions is called a random experiment.

∴ Sample Space: The collection of all possible outcomes of a random experiment is called sample space.

e.g. when we roll a fair dice

$$S = \{1, 2, 3, 4, 5, 6\}$$

Outcome: The result obtained from an experiment or from an event is called outcome.

Event: Any part or subset of the sample space is called event.

✓ Simple Event: Any event having only one sample point is called simple event.

✓ Compound Event: Any event having two (25) or more sample points is called compound event.

Sure Event: Any event having all possible outcomes of the sample space is called sure event.
i.e. $P(S) = 1$.

Impossible Event: Any event having no sample point is called impossible event.
 $P(\phi) = 0$

✓ Mutually Exclusive Events: Two events A and B are said to be mutually exclusive events, if they can not occur both at a time.

Equally likely Events: Two events A and B are said to be equally likely, if A is as likely to occur as B.

Exhaustive Events / Collectively Exhaustive Events:

Two events A and B are said to be collectively exhaustive, if their union is the complete sample space.

Q: Describe the Rules of Counting.

Q: Define the rules of Counting?
Ans: There are three rules of counting.

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(i) Permutation:

(ii) Combination.

(iii) Multiplication.

Permutations: The different ways in which the no. of objects can be arranged in a definite order is called permutation.

$${}^n P_r = \frac{n!}{(n-r)!}$$

Combination: The different ways in which the no. of objects can be arranged in any order are called combinations.

$${}^n C_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

• Multiplication Rule: If two random experiments have "m" and "n" outcomes resp. Then there will be "m x n" outcomes.

Independent Events: Two events A and B are said to be independent events, if the occurrence of one does not effect upon the occurrence of other.

Note: In this case $P(A \cap B) = P(A) \cdot P(B)$

□ Dependent Events: Two events A and B are said to be dependent if the occurrence of one affects upon the occurrence of other. (27)

Note: In this case $P(A \cap B) \neq P(A) \cdot P(B)$

▷ Conditional probability If A and B are any two events, then the probability of any one of them, while the other has already occurred is called conditional probability.

$$\text{i.e. } P(A/B) = \frac{P(A \cap B)}{P(B)} \quad \text{iff } P(B) \neq 0$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)} \quad \text{iff } P(A) \neq 0$$

Note: Addition Laws of Probability

Case I: When A and B are mutually Exclusive events, then

$$P(A \cup B) = P(A) + P(B)$$

Case II: When A and B are not mutually Exclusive events, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Multiplication Law of Probability

Case I: When A and B are Independent

$$P(A \cap B) = P(A) \cdot P(B)$$

Case II: When A and B are dependent

$$P(A \cap B) = P(B) \cdot P(A/B) \quad \text{iff } P(B) \neq 0$$

Variable: Any characteristic that varies either in quantity or in quality from one individual to other is called variable.

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eg.

- I The no. of persons per house in R.K.
- II The no. of rooms per house in R.K.

Random Variable: Any variable whose values are determined by the outcomes of a random experiment is called random variable.

eg.

- I The no. of accidents occurred per day on G.T. Road.
- II The no. of flowers per branch on a plant.

Constant: Any value which does not change but remains fixed is called a constant.

Discrete Variable: Any variable having specified or countable no. of values is called discrete variable.

eg.

- I The no. of students per class in K.P.K.
- II The no. of rooms per house in R.K.

Continuous Variable: Any variable having measurable values in an interval is called continuous variable.

eg.

The height, weight, speed, temperature, time and age etc. are examples of continuous variable.

Quantitative Variable: Any variable which 29 changes only in quantity from one individual to other is called quantitative variable.

e.g.

- i Per capita income in RPK.
- ii Import and export rate in Pakistan.

Qualitative variable (Attribute)

Any variable which changes only in quality from one individual to other, is called qualitative variable or Attribute.

e.g.

- i Blindness
- ii Intelligence.
- iii Beauty etc.

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Q: What is discrete probability distribution?
write down its properties also.

Ans: Let x is a discrete Random variable having values $x_1, x_2, x_3, \dots, x_n$ with respective probabilities $p(x_1), p(x_2), p(x_3), \dots, p(x_n)$. Then the arrangement of the values along with their respective probabilities in tabular form is called probability dist.

e.g.

$x:$	x_1	x_2	x_3	\dots	x_n	total.
$p(x):$	$p(x_1)$	$p(x_2)$	$p(x_3)$	\dots	$p(x_n)$	$= 1$

A probability dist. has following two properties.

$$\sum 0 \leq p(x) \leq 1 \quad \text{for each } x.$$

(30)

$$\sum p(x) = 1$$

Q: Define Continuous probability density function (pdf)

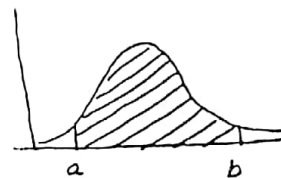
Ans: A function with values $f(x)$ is called a probability density function (pdf) for the continuous random variable x .

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i The total Area under its curve is equal to 1

ii The area under the curve b/w any two ordinates $x=a$ and $x=b$ is given as:

$$P(a \leq x \leq b)$$



Q: What are the properties of probability density function?

Ans: Each probability density function has two properties:

i The function is non negative.

ii The total Area under the curve is unity.

Q: What do you mean by mathematical expectation of a random variable?

Ans: Let " x " is a discrete random variable

having values $x_1, x_2, x_3, \dots, x_n$ with respective

probabilities $p(x_1), p(x_2), p(x_3), \dots, p(x_n)$. Then

expected value or mean value of x is given as:

$$E(x) = x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3) + \dots + x_n p(x_n)$$

$$E(x) = \sum x p(x)$$

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Similarly as

$$E(x^2) = \sum x^2 p(x)$$

$$E(x^3) = \sum x^3 p(x)$$

⋮

$$E(x^r) = \sum x^r p(x)$$

Q: What are laws of expectation?

If x, y are two variables, where a, b are Constt.

i. $E(a) = a$ i.e. expected value of a Constt. is Constt. itself.

$$ii. E(x+a) = E(x) + a$$

$$iii. E(ax+b) = aE(x) + b$$

$$iv. E(x+y) = E(x) + E(y)$$

$$v. E(x-\mu) = 0$$

$$vi. E(xy) = E(x) \cdot E(y), \text{ if } x \text{ and } y \text{ are independent variables.}$$

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□ Distribution Function or Cumulative Dist. function.

The distribution function of a random variable x gives the probability of the event that x takes a value less than or equal to a specified value of x .

$$i.e. F(x) = P(X \leq x)$$

It has following properties:

$$i. F(-\infty) = 0, F(+\infty) = 1$$

ii. $F(x)$ is a non decreasing function of x

$$i.e. F(x_1) \leq F(x_2) \text{ if } x_1 < x_2$$

$F(x)$ is continuous at least on right of each x .

Joint distribution. The distribution of two or more random variables which are studied simult. when an experiment is performed is called joint distribution.

Bivariate Probability dist. The probability dist. of two random variables, which are studied simult. when an experiment is performed is called bivariate probability dist.

Marginal Probability dist.

The probability dist. obtained for any individual variable, from joint probability dist. is called marginal probability dist.

Independence of two random variables.

Two random variables x & y are said to be independent, if their joint dist. is the product of their marginal probability dist.

$$\text{i.e. } f(x, y) = g(x) \cdot h(y)$$

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Conditional Probability Function.

Let x and y are two random variables. then the probability of any one of them, while the other has already occurred is given as:

$$f(x/y) = \frac{f(x,y)}{h(y)} \quad \text{iff } h(y) \neq 0$$

$$f(y/x) = \frac{f(x,y)}{g(x)} \quad \text{iff } g(x) \neq 0$$

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Moments Generating Function *

The moment generating function of a random variable x about origin is defined as the expected value of the r.v. e^{tx} , where t is a real variable lying in a neighbourhood of zero.

$$\text{i.e. } m(t) = E(e^{tx})$$

$$= \sum e^{tx} p(x) \quad \text{in case of discrete r.v.}$$

$$= \int e^{tx} f(x) dx \quad \text{in case of continuous r.v.}$$

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